



Aggregating Public and Private Goods

Ben Glasner



Content

- I. **Aggregating Private Goods**
- II. **Aggregating Public Goods**

Aggregating Private Goods

- > **Aggregate market for private goods:**
 - **Excludable**
 - **Rivalrous**
 - **For two people to consume, you need two goods**
 - **Horizontal Summation**

Aggregating Private Goods

- > George, John, Paul, and Ringo constitute the entire market for yellow submarines. Their respective demands are:

$$Q_G = 30 - 10P$$

$$Q_J = 20 - 5P$$

$$Q_P = 10 - 2P$$

$$Q_R = 6 - 1P$$

- > If we wanted to talk about the total quantity of yellow submarines demanded at a given price, what would we do?

Aggregating Private Goods

$$Q_G = 30 - 10P$$

$$Q_J = 20 - 5P$$

$$Q_P = 10 - 2P$$

$$Q_R = 6 - 1P$$

$$P \geq 5, \quad Q_{agg} =$$

$$5 > P \geq 4, \quad Q_{agg} =$$

$$4 > P \geq 3, \quad Q_{agg} =$$

$$3 > P, \quad Q_{agg} =$$

Aggregating Private Goods

$$Q_G = 30 - 10P$$

$$Q_J = 20 - 5P$$

$$Q_P = 10 - 2P$$

$$Q_R = 6 - 1P$$

$$P \geq 5,$$

$$Q_{agg} = 6 - 1P$$

$$5 > P \geq 4,$$

$$Q_{agg} = 6 - 1P + 10 - 2P = 16 - 3P$$

$$4 > P \geq 3,$$

$$Q_{agg} = 16 - 3P + 20 - 5P = 36 - 8P$$

$$3 > P,$$

$$Q_{agg} = 36 - 8P + 30 - 10P = 66 - 18P$$

Aggregating Private Goods

$$Q_G = 30 - 10P$$
$$Q_P = 10 - 2P$$

$$Q_J = 20 - 5P$$
$$Q_R = 6 - 1P$$

P	George	John	Paul	Ringo	Aggregated Market
0					
1					
2					
3					
4					
5					

Aggregating Private Goods

$$Q_G = 30 - 10P$$
$$Q_P = 10 - 2P$$

$$Q_J = 20 - 5P$$
$$Q_R = 6 - 1P$$

P	George	John	Paul	Ringo	Aggregated Market
0	30	20	10	6	66
1	20	15	8	5	48
2	10	10	6	4	30
3	0	5	4	3	12
4	0	0	2	2	4
5	0	0	0	1	1

Aggregating Private Goods

George:

$$Q_G = 30 - 10P$$

$$P = \frac{30 - Q_G}{10}$$

Paul:

$$Q_P = 10 - 2P$$

$$P = \frac{10 - Q_P}{2}$$

John:

$$Q_J = 20 - 5P$$

$$P = \frac{20 - Q_J}{5}$$

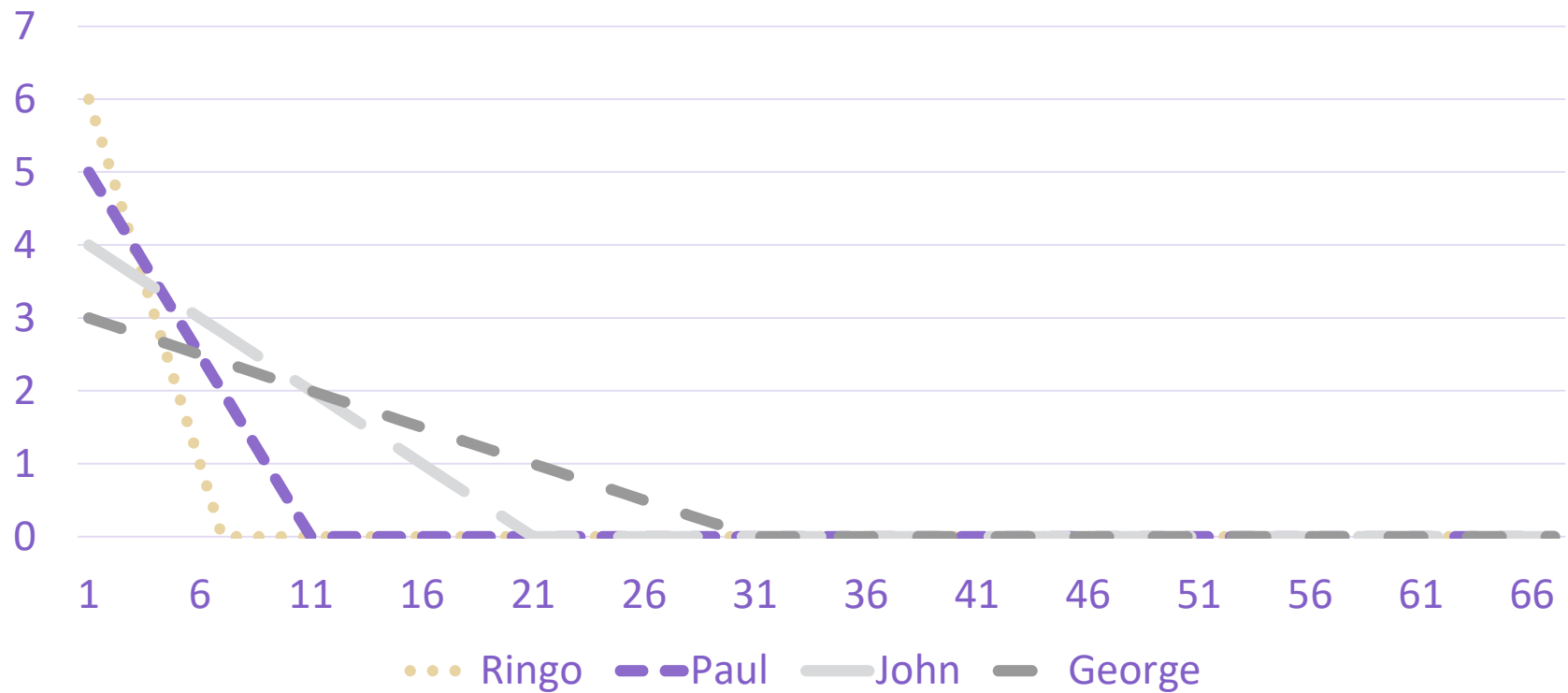
Ringo:

$$Q_R = 6 - 1P$$

$$P = 6 - Q_R$$

Aggregating Private Goods

Private Market Aggregation (Horizontal) of Demand



Aggregating Private Goods

$$1 > Q \geq 0, \quad Q_{agg} = 6 - 1P \quad P = 6 - Q_{agg}$$

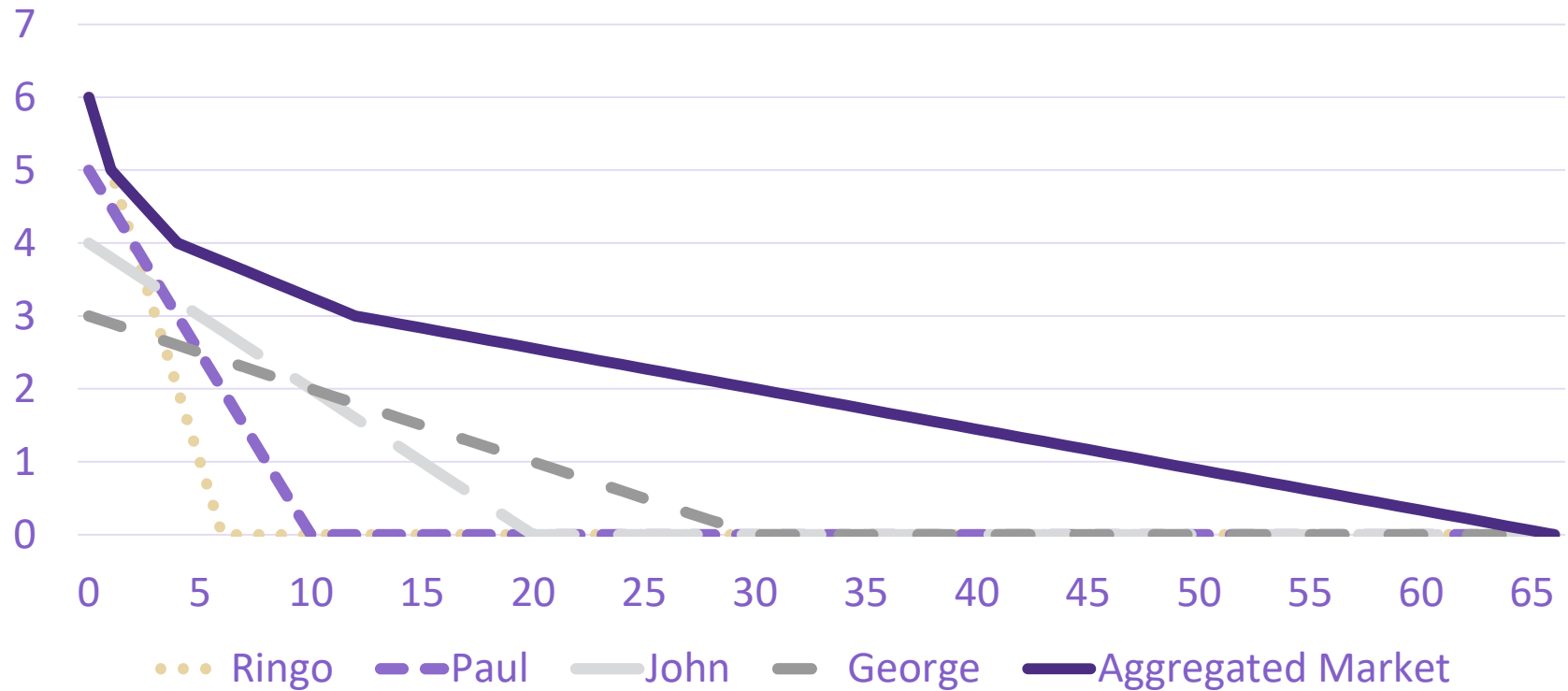
$$4 > Q \geq 1, \quad Q_{agg} = 16 - 3P \quad P = \frac{16 - Q_{agg}}{3}$$

$$12 > Q \geq 4, \quad Q_{agg} = 36 - 8P \quad P = \frac{36 - Q_{agg}}{8}$$

$$Q \geq 12, \quad Q_{agg} = 66 - 18P \quad P = \frac{66 - Q_{agg}}{18}$$

Aggregating Private Goods

Private Good, Horizontal Aggregation



Aggregating Public Goods

- > **Aggregate market for Public goods:**
 - **Non-excludable**
 - **Non-rivalrous**
 - **For two people to consume, you only need one unit**
 - **Vertical Summation of individual demand**

Aggregating Public Goods

- > **Aggregate market for public goods:**
 - I do not need to buy a public good to enjoy it, I just need it to exist
 - To get a sense of the aggregate demand we use “Willingness to Pay”
 - > What is the most a consumer would be willing to pay to purchase the next unit
 - We want to get a sense of how much benefit is created for every additional unit of the good

Aggregating Public Goods

- > George, John, Paul, and Ringo constitute the entire market for public art:

$$Q_G = 30 - 10P$$

$$Q_J = 20 - 5P$$

$$Q_P = 10 - 2P$$

$$Q_R = 6 - 1P$$

- > If we wanted to talk about the marginal willingness to pay for each additional unit of art, we use a vertical summation

Aggregating Public Goods

George:

$$Q_G = 30 - 10P$$

$$P_G = \frac{30 - Q_G}{10}$$

John:

$$Q_J = 20 - 5P$$

$$P_J = \frac{20 - Q_J}{5}$$

Paul:

$$Q_P = 10 - 2P$$

$$P_P = \frac{10 - Q_P}{2}$$

Ringo:

$$Q_R = 6 - 1P$$

$$P_R = 6 - Q_R$$

Aggregating Public Goods

$$P_G = \frac{30 - Q_G}{10}$$

$$P_J = \frac{20 - Q_J}{5}$$

$$P_P = \frac{10 - Q_P}{2}$$

$$P_R = 6 - Q_R$$

Q	George	John	Paul	Ringo	(Marginal) WTP by Vertical Summation
0					
5					
10					
15					
20					
25					

Aggregating Public Goods

$$P_G = \frac{30 - Q_G}{10}$$

$$P_J = \frac{20 - Q_J}{5}$$

$$P_P = \frac{10 - Q_P}{2}$$

$$P_R = 6 - Q_R$$

Q	George	John	Paul	Ringo	(Marginal) WTP by Vertical Summation
0	3	4	5	6	18
5	2.5	3	2.5	1	9
10	2	2	0	0	4
15	1.5	1	0	0	2.5
20	1	0	0	0	1
25	0.5	0	0	0	0.5

Aggregating Public Goods

Public Good, Vertical Aggregation

